

A NOTE ON sg^* CONTINUOUS MAPPINGS IN SOFT TOPOLOGICAL SPACES

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Abstract: The aim of this paper is to introduce sg^* closed set in a Soft topological space and to study some of its properties. Then sg^* continuous mapping and irresolute mapping are introduced and some of its properties are studied. The concept sg^* open, sg^* closed mappings and sg^* homeomorphism are introduced and their properties are studied.

Key-Words: sg^* continuous mapping , irresolute mapping, sg^* homeomorphism

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1 . INTRODUCTION

The theory of soft sets gives a vital mathematical tool for handling uncertainties and vague concepts. In the year 1999, Molodtsov[1] initiated the study of soft sets. Soft set theory has been applied in several directions. Following this Maji, Biswas, and Roy[7,8] discussed soft set theoretical operations and gave an application of soft set theory to a decision making problem. Recently Muhammad Shabir and Munazza Naz introduced the notion of soft topology[10] and established that every soft topology induces a collection of topologies called the parametrized family of topologies induced by the soft topology. Several mathematicians published papers on applications of soft sets and soft topology[1,2,6,11,12,1]. Soft sets and soft topology have applications to data mining, image processing, decision making problems, spatial modeling and neural patterns[3,4,5,7]. In this paper, the concept sg^* closed set is introduced in soft topological space and the concept of sg^* continuous mapping and sg^* irresolute mapping are introduced and some of their properties are studied. Further the concept sg^* open , sg^* closed mappings and sg^* homeomorphism are introduced and some of their basic soft topological properties are investigated. Finally the concept of slightly sg^* continuous mapping is introduced and studied some of its basic concepts.

2. PRELIMINARIES

2.1 Definition A soft set (A, E) is called sg^* closed in a soft topological space $(X, \tilde{\tau}, E)$ of $cl(A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft g open in $\tilde{X}$.

2.2.1 Let $X = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$ and

$\tilde{\tau} = \{\emptyset, \tilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E), (A_6, E), (A_7, E)\}$ where

$$(A_1, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\})\}, \quad (A_2, E) = \{(b_1, \{a_2\}), (b_2, X)\}$$

$$(A_3, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_2, a_3\})\}, \quad (A_4, E) = \{(b_1, \{a_1, a_3\}), (b_2, X)\},$$

$$(A_5, E) = \{(b_1, \emptyset), (b_2, \{a_1\})\} \quad (A_6, E) = \{(b_1, \emptyset), (b_2, \{a_2, a_3\})\} \text{ and}$$

$$(A_7, E) = \{(b_1, \emptyset), (b_2, X)\}.$$

Clearly $(A, E) = \{(b_1, \{a_1, a_3\}), (b_2, \{a_3\})\}$ is sg^* closed in $(X, \tilde{\tau}, E)$.

since for (A, E) there exists a soft g open set $(U, E) = \{(b_1, \{a_1, a_3\}), (b_2, \{a_2, a_3\})\}$ such that $cl(A, E) \subseteq (U, E)$.

2.1 Theorem

Every soft closed set is sg^* closed in a soft topological space $(X, \tilde{\tau}, E)$.

3. sg^* CONTINUOUS MAPPINGS

3.1 Definition

A soft mapping $f: \tilde{X} \rightarrow \tilde{Y}$ is called sg^* continuous if $f^{-1}(U, E)$ is sg^* closed in $(X, \tilde{\tau}, E)$ for every soft closed set (U, E) of $(Y, \tilde{\omega}, E)$.

3.2. Theorem

Let $f: \tilde{X} \rightarrow \tilde{Y}$ be a soft mapping from soft topological space $(X, \tilde{\tau}, E)$ into a soft topological space $(Y, \tilde{\omega}, E)$. Then the following statements are equivalent.

- i) $f: \tilde{X} \rightarrow \tilde{Y}$ is sg^* continuous.
- ii) The inverse image of each soft open set in $\tilde{Y}$ is sg^* open in $\tilde{X}$.
- iii) For each soft subset $(A, E) \subseteq (Y, \tilde{\omega}, E)$ $sg^*cl(f^{-1}(A, E)) \subseteq f^{-1}cl(A, E)$.

i v) For each soft subset $(B, E) \in (X, \tilde{r}, E)$ $f(sg^*cl(B, E)) \subseteq cl(f(B, E))$.

Proof (i) $\rightarrow$ (ii) follows from 3.1 Definition.

(i) $\rightarrow$ (iii)

Let (A, E) be a soft subset of $(Y, \tilde{\omega}, E)$. By 3.2.1 Definition $f^{-1}cl(A, E)$ is a sg^* closed set containing $f^{-1}(A, E)$ and $sg^*cl(f^{-1}(A, E)) \subseteq f^{-1}cl(A, E)$.

(iii) $\rightarrow$ (iv)

Let $(B, E) \in (Y, \tilde{r}, E)$, then $f(B, E) \in (Y, \tilde{\omega}, E)$ Hence from (iii) $sg^*cl(f^{-1}(f(B, E))) \subseteq f^{-1}(cl(A, E))$. Therefore $f(sg^*cl(B, E)) \subseteq clf(B, E)$.

(iv) $\rightarrow$ (i)

Let (U, E) be a soft closed set in $\tilde{Y}$. Then by (iv)

$f(sg^*cl(f^{-1}(U, E))) \subseteq cl(f(f^{-1}(U, E)))$. Hence $sg^*cl(f^{-1}(U, E)) \subseteq f^{-1}(U, E)$. Therefore $f^{-1}(U, E)$ is a sg^* closed set in $\tilde{X}$.

3.3 Theorem

Let $f: \tilde{X} \rightarrow \tilde{Y}$ be a soft continuous mapping from $\tilde{X}$ into $\tilde{Y}$. Then it is sg^* continuous.

Proof

(i) $\rightarrow$ (ii) follows from 3.1 Definition.

(i) $\rightarrow$ (iii)

Let (A, E) be a soft subset of $(Y, \tilde{\omega}, E)$. By 3.1 Definition $f^{-1}(cl(A, E))$ is a sg^* closed set containing $f^{-1}(A, E)$ and $sg^*cl(f^{-1}(A, E)) \subseteq f^{-1}(cl(A, E))$.

(iii) $\rightarrow$ (iv)

Let $(B, E) \in (X, \tilde{r}, E)$. Then $f(B, E) \in (Y, \tilde{\omega}, E)$. Hence from (iii) $sg^*cl(f^{-1}(f(B, E))) \subseteq f^{-1}(clf(B, E))$. Therefore $f(sg^*cl(B, E)) \subseteq clf(B, E)$.

(iv) $\rightarrow$ (i)

Let (U, E) be a soft closed set in $\tilde{Y}$. Then by (iv)

$f(sg^*cl(f^{-1}(U, E))) \subseteq cl(f(f^{-1}(U, E)))$. Hence $sg^*cl(f^{-1}(U, E)) \subseteq f(U, E)$.

Therefore $f^{-1}(U, E)$ is a sg^* closed set in $\tilde{X}$.

3.4 Theorem

Let $f: \tilde{X} \rightarrow \tilde{Y}$ be a soft continuous mapping from $\tilde{X}$ into $\tilde{Y}$. Then it is sg^* continuous.

Proof

Let (A, E) be any soft closed set in $\tilde{Y}$. Then $f^{-1}(A, E)$ is soft closed in $\tilde{X}$. Therefore by 2.1 Theorem, $f^{-1}(A, E)$ is sg^* closed in $\tilde{X}$.

3.5 Example

The following example shows that the converse of the above 3.2.2 Theorem need not be true.

Let $X = \{a_1, a_2, a_3\}, Y = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$ and

$$\tilde{r}_1 = \{\tilde{\emptyset}, \tilde{X}, (B_1, E), (B_2, E), (B_3, E), (B_4, E), (B_5, E)\}$$

$\tilde{r}_1 = \{\tilde{\emptyset}, \tilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E), (A_6, E)\}$ be two soft topological spaces over X and Y respectively. Then $(B_1, E), (B_2, E), (B_3, E), (B_4, E), (B_5, E)$ are soft sets over X and $(A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E)$ are soft sets over Y defined as follows:

$$\begin{aligned} (A_1, E) &= \{(b_1, \{a_2, a_3\}), (b_2, \{a_1, a_3\})\}, & (A_2, E) &= \{(b_1, \{a_3\}), (b_2, \{a_1\})\}, \\ (A_3, E) &= \{(b_1, \{a_2\}), (b_2, \{a_3\})\}, & (A_4, E) &= \{(b_1, \{a_3\}), (b_2, \emptyset)\}, \\ (A_5, E) &= \{(b_1, X), (b_2, \{a_1, a_3\})\}, & (A_6, E) &= \{(b_1, \{a_2, a_3\}), (b_2, \{a_3\})\}, \\ (B_1, E) &= \{(b_1, \{a_2\}), (b_2, \{a_1\})\} & (B_2, E) &= \{(b_1, \{a_3\}), (b_2, \{a_1, a_3\})\}, \\ (B_3, E) &= \{(b_1, \{a_2, a_3\}), (b_2, \{a_1, a_2\})\}, & (B_4, E) &= \{(b_1, X), (b_2, \{a_1, a_2\})\}, \\ \text{and } (B_5, E) &= \{(b_1, \emptyset), (b_2, \{a_1\})\}. \end{aligned}$$

Let $f: \tilde{X} \rightarrow \tilde{Y}$ be a soft mapping defined by $f(a_1) = a_1$, $f(a_2) = a_3$, and $f(a_3) = a_2$. Then f is sg^* continuous map but not soft continuous. Since $f^{-1}(A_1, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_1, a_2\})\}$,

$$f^{-1}(A_2, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\})\}, \quad f^{-1}(A_3, E) = \{(b_1, \{a_3\}), (b_2, \{a_2\})\},$$

$$f^{-1}(A_4, E) = \{(b_1, \{a_2\}), (b_2, \emptyset)\}, \quad f^{-1}(A_5, E) = \{(b_1, X), (b_2, \{a_1, a_2\})\},$$

$$f^{-1}(A_6, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_2\})\} \quad \text{are } sg^* \text{ open sets in } \tilde{r}_1 \text{ but}$$

$$f^{-1}(A_3, E), f^{-1}(A_4, E), f^{-1}(A_5, E), f^{-1}(A_6, E) \text{ are not soft open sets in } \tilde{r}_1.$$

3.6 Theorem

If $f: \tilde{X} \rightarrow \tilde{Y}$ is a sg^* continuous mapping from $\tilde{X}$ into $\tilde{Y}$ then f is soft g continuous.

Proof Let (A, E) be any soft closed set in $\tilde{Y}$. Then $f^{-1}(A, E)$ is sg^* closed in $\tilde{X}$. Therefore by 2.1 Theorem $f^{-1}(A, E)$ is soft g closed in $\tilde{X}$.

3.7 Definition

A soft mapping $f: \tilde{X} \rightarrow \tilde{Y}$ called sg^* irresolute if $f^{-1}(U, E)$ is sg^* closed in $\tilde{X}$ for every sg^* closed set of $(Y, \tilde{\omega}, E)$.

3.8 Remark

A soft mapping $f: \tilde{X} \rightarrow \tilde{Y}$ is sg^* irresolute if and only if the inverse image of every sg^* open set in $(Y, \tilde{\omega}, E)$ is sg^* open in $\tilde{X}$.

3.9 Theorem If $f: \tilde{X} \rightarrow \tilde{Y}$ and $h: \tilde{Y} \rightarrow \tilde{Z}$ are any two soft mappings then

- i) $h \circ g$ is sg^* continuous if h is soft continuous and f is sg^* continuous.
- ii) $h \circ g$ is sg^* continuous if h is sg^* continuous and g is sg^* irresolute.
- iii) $h \circ g$ is sg^* irresolute if both g and h are sg^* irresolute.

Proof

(i) Let (U, E) be a soft closed set in $\tilde{Z}$. Then $h^{-1}(U, E)$ is soft closed in $\tilde{Y}$ and $g^{-1}(h^{-1}(U, E)) = h \circ g(U, E)$ is sg^* closed in $\tilde{X}$.

(ii) Let (U, E) be a soft closed set in $\tilde{Z}$. Then $h^{-1}(U, E)$ is sg^* closed in $\tilde{Y}$ and $g^{-1}(h^{-1}(U, E)) = h \circ g(U, E)$ is sg^* closed in $\tilde{X}$.

(iii) Let (U, E) be a sg^* closed set in $\tilde{Z}$. Then $h^{-1}(U, E)$ is sg^* closed in $\tilde{Y}$ and $g^{-1}(h^{-1}(U, E)) = h \circ g(U, E)$ is sg^* closed in $\tilde{X}$.

3.10 Theorem

A soft mapping $f: \tilde{X} \rightarrow \tilde{Y}$ is sg^* irresolute if and only if for every soft subset (U, E) of $\tilde{X}$, $g(sg^* cl(U, E)) \subseteq sg^* cl(g(U, E))$.

Proof Let g be a sg^* irresolute mapping and (U, E) be a soft subset in $\tilde{X}$. Then $sg^* cl(g(U, E))$ is sg^* closed set in $\tilde{Y}$. Hence $g^{-1}(sg^* cl(g(U, E)))$ is sg^* closed in $\tilde{X}$ and $(U, E) \subseteq g^{-1}(g(U, E)) \subseteq g^{-1}(sg^* cl(g(U, E)))$.

Therefore

$$sg^* cl(U, E) \subseteq g^{-1}(sg^* cl(g(U, E))), \text{ hence } g(sg^* cl(U, E)) \subseteq sg^* cl(g(U, E)).$$

Conversely, suppose that (U, E) is sg^* closed in $\tilde{Y}$.

Therefore

$$g(sg^* cl(g^{-1}(U, E))) \subseteq (sg^* cl(g(g^{-1}(U, E)))) = sg^* cl(U, E) = (U, E). \text{ Hence } sg^* cl(g^{-1}(U, E)) \subseteq g^{-1}(U, E).$$

4. sg^* HOMEOMORPHISMS

4.1 Definition

A soft mapping $f: \tilde{X} \rightarrow \tilde{Y}$ is called sg^* open if $g(U, E)$ of each soft open set (U, E) in $(X, \tilde{r}, E)$ is sg^* open in $(Y, \tilde{\omega}, E)$.

4.2 Definition

A soft mapping $f: \tilde{X} \rightarrow \tilde{Y}$ is called sg^* closed if $g(U, E)$ of each soft closed set (U, E) in $(X, \tilde{r}, E)$ is sg^* closed in $(Y, \tilde{\omega}, E)$.

4.3 Theorem

Let the soft mappings $f: \tilde{X} \rightarrow \tilde{Y}$ and $g: \tilde{Y} \rightarrow \tilde{Z}$ be bijective. If $g \circ f: \tilde{X} \rightarrow \tilde{Z}$ is soft continuous and $f: \tilde{X} \rightarrow \tilde{Y}$ is soft continuous and $f: \tilde{X} \rightarrow \tilde{Y}$ is sg^* closed then $g: \tilde{Y} \rightarrow \tilde{Z}$ is sg^* continuous.

Proof

Let (U, E) be the soft closed set in $\tilde{Z}$. Since $g \circ f: \tilde{X} \rightarrow \tilde{Z}$ is soft continuous, then $f^{-1}(g^{-1}(U, E)) = (g \circ f)^{-1}(U, E)$ is soft closed set in $\tilde{X}$. Since $f: \tilde{X} \rightarrow \tilde{Y}$ is sg^* closed, then $f(f^{-1}(g^{-1}(U, E))) = g^{-1}(U, E)$ is sg^* closed set in $\tilde{Y}$.

4.5 Theorem

A soft mapping $f: \tilde{X} \rightarrow \tilde{Y}$ is a sg^* open iff if $f(ikt(B, U)) \subseteq sg^*ikt(f(B, E))$ for every soft subset (B, E) of $\tilde{X}$.

Proof

Let $f: \tilde{X} \rightarrow \tilde{Y}$ be sg^* open and (B, E) be a soft subset of $\tilde{X}$, then $ikt(B, U)$ is a soft open set in $\tilde{X}$. Hence $f(ikt(B, E)) = sg^*ikt(f(ikt(B, E)))$.

Conversely, Let (G, E) be a soft open set in $\tilde{X}$. $f(G, E) = f(ikt(G, E)) \subseteq sg^*ikt(f(G, E))$, which implies $f(G, E) \subseteq sg^*ikt(f(G, E))$. Hence $f(G, E)$ is a sg^* open in $\tilde{Y}$.

4.6 Definition

If a soft mapping $f: \tilde{X} \rightarrow \tilde{Y}$ is sg^* continuous bijective and f^{-1} is sg^* continuous then f is said to be sg^* homeomorphism from $(X, \tilde{r}, E)$ in to $(Y, \tilde{w}, E)$.

4.7 Theorem

Let $f: \tilde{X} \rightarrow \tilde{Y}$ be the soft bijective mapping. Then the following statements are equivalent: .
Since f is sg^* open map,

- i) $f^{-1}: \tilde{Y} \rightarrow \tilde{X}$ is sg^* continuous.
- ii) f is sg^* open .
- iii) f is sg^* closed.

Proof

(i) $\rightarrow$ (ii) Let (U, E) be any soft open set in $\tilde{X}$. Since $f^{-1}: \tilde{Y} \rightarrow \tilde{X}$ is sg^* continuous, therefore $(f^{-1})^{-1}(U, E) = f(U, E)$ is sg^* open in $\tilde{Y}$.

(ii) $\rightarrow$ (iii) Let (B, E) be any soft closed set in $\tilde{X}$, then $\tilde{X} - (B, E)$ is soft open set in $\tilde{X}$. Since f is sg^* open map, $f(\tilde{X} - (B, E))$ is sg^* open in $\tilde{Y}$. But $f(\tilde{X} - (B, E)) = \tilde{Y} - f(B, E)$, implies $f(B, E)$ is sg^* closed in $\tilde{Y}$.

(iii) $\rightarrow$ (i) Let (B, E) be any soft closed set in $\tilde{X}$. Then $(f^{-1})^{-1}(U, E) = f(U, E)$ is sg^* closed in $\tilde{Y}$. Therefore $f^{-1}: \tilde{Y} \rightarrow \tilde{X}$ is sg^* continuous.

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