Time-Cost Trade-off Analysis for Building Construction Projects

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Abstract:

In Construction planning the major problem is reducing the time- cost and it developed the time- cost trade-off (TCTO) problem moments. The time and cost is always identified with each other and if we assume time as precious as plutocrat (time value of money) TVOM also the dealing with these type of problem is must be easy. In moment's competitive business terrain, delivering systems in the least possible time, with maximum quality and minimal cost has got a critical issue for design directors. Design time crashing plays an important part in design operation determining which conditioning duration to crash to complete the design in the quested time. TCTO problem in design operation have been traditionally answered by two distinctive approaches like CTM, PERT, Heuristic styles and so on. The relative analysis between mathematically (Linear Programming Method/ Traditional) and CTM is done by using an illustration.

Keywords: Project management; Time-cost trade-off, CPM, Optimization, Network Analysis.

- **INTRODUCTION**

Definitions of Time-Cost Trade-off

One important extension to the basic network analysis technique relates to project cost/ project time tradeoff. In this extension to the basic method we assume that, for each activity, the completion time can be reduced (within limits) by spending more money on the activity.

Normal time: - It is time taken by the **project** without any delay in any activity of the project. Normal time of the project does not contain crashing of any activities. Crash cost: - Crash cost is the cost associated when the project is completed with crash time of the project.

A procedure for determining the optimal project time is to determine the normal completion time for each critical path activity and a crash time. The crash time is the shortest time in which an activity can be completed. The direct **costs** then are calculated for the normal and crash times of each activity.

Reducing project duration can be done by adjusting overlaps between activities or by reducing activities' duration. What is the reason for an increase in direct cost as the activity duration is reduced. A simple case arises in the use of overtime work. By scheduling weekend or evening work, the completion time for an activity as measured in calendar days will be reduced. However, extra wages must be paid for such overtime work, so the cost will increase. Also, overtime work is more prone to accidents and quality problems that must be corrected, so costs may increase. The activity duration can be reduced by one of the following actions:

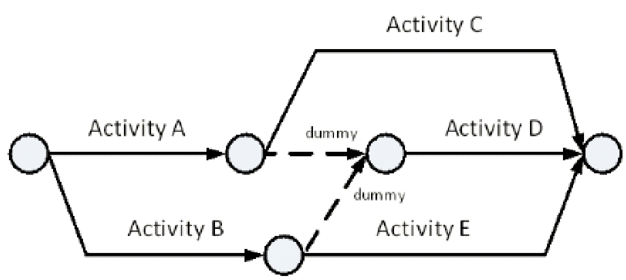
- Applying multiple-shifts work.
- Working extended hours (over time).
- Offering incentive payments to increase the productivity.
- Working on weekends and holidays.
- Using additional resources.
- Using materials with faster installation methods.

- Using alternate construction methods or sequence.

Garvin (1984) proposes several dimensions of quality from which we select conformance (match with specifications) as opposed to perceived quality which is the most subjective dimension. Conformance makes quality a measure variable whereas perceived quality is a useful concept in the early design stages of a new product. As an example, smell intensity of car interior is a component of perceived quality. A thorough analysis of perceived quality may be found in Stylidis et al. (2015). For non innovative projects that are under the scope of this survey, quality as conformance to customer specifications and to technical requirements is the relevant definition. Customer specifications involve requirements related to aesthetics (fits and finishes) or to the utilization of high quality components and raw materials to ensure a longer product life.

For applying a network model in civil engineering practice, it must be suitable for handling two features in Consideration of scheduling.

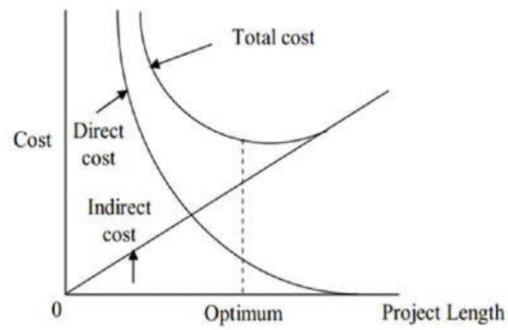
(a) The first one is the possibility of changing process durations depending on their start times. This is the key to apply calendar.
(b) The second one is using maximal constraints for activities and connections. This is useful and important in practice. In linear programming method, it is possible to give only minimal constraints. For applying maximal constraint, it must be converted by multiply the assumption with (*-1*). It effects negative process time and turning back arc which generates loops.
In case of this constraint, there is no limit to apply the solution for the models either activity on edge (AOE) or activity on node (AON). Here, notations are related to the model AOE.



The problem is known as time/cost trade-off problem in the project management literature.. In Khang and Myint (1999), model proposed by Babu and Suresh (1996) was applied to an actual cement factory construction project. Time cost quality trade-off has been thereafter researched from a mathematical point of view using heavy mathematical programming methods and software. This paper focuses more on the decision making aspect of the trade-off analysis and uses an easily available, user friendly Microsoft Excel Solver Add-in for analysis. This research also validates with real data most of the findings by Khang and Myint in their original work. The importance of time/cost trade-offs in projects have been recognized since the development of the critical path method (CPM) in the late 1950s. Sustainable project management requires the resources to be used in an economical and sustainable way . Therefore, the duration of project activities can be treated as discrete non-increasing functions of the cost. This results in the discrete time/cost trade-off problem (DTCTP) . Harvey and Patterson and Hindelang and Muth first proposed the DTCTP, which is a special case of the multi-mode resource-constrained project scheduling problem The paper is organized as follows: In the next section we briefly define the deterministic time-cost trade-off Problem.

- Time-cost Trade off Problem (TCTP)

To start scheduling the project, an arbitrary date should be set for the start event in the project's network diagram. Usually this value is set to zero. Scheduling computation consists of two tracking through the network, forward tracking and backward tracking. The earliest possible date to start each activity is determined by forward tacking, and the latest possible date to finish the ones is obtained by backward tracking. The Earliest possible date to start an activity (ES) is the earliest date that all of its predecessor activities have been finished. The Latest possible date to Finish an activity (LF) is the latest date that none of its successor activities would be started. If an activity finishes after this date, it will have effect on the start dates of its successor activities.



The PERT/CPM methods of time-cost trade-offs is concerned with determining how much (if any) to crash each of the activities in order to reduce the anticipated duration of the project to a desired value.
The major reasons which make the project managers interested in using the crashing models are:

- Avoiding unfavorable weather conditions
- Early commissioning and operation
- Improving the project cash flow
- Compensating the delays
- Early utilization

Quality issues implied by overlapping are resolved using rework to correct design flaws resulting from partial information. Thus for such projects, quality is implicitly taken into account with rework (see for instance Lin et al., 2010 for a recent contribution). Most of Time Cost Quality Trade-off Problems(TCQTP) are modelled in a similar way to Time Cost Trade-off Problems (TCTP) that have been extensively studied in the literature related to deterministic project scheduling (see Weglarz et al., 2011 for a recent survey). The time and cost parameters of a construction project have been identified as major factors of the decision making process. In critical-path method (CPM), generally, the objective is to establish a minimum project cost with reasonable time schedule based on realistic assumptions. The primary impact of project timing is money and project time is thus an equally essential factor. Since cost can be expressed as a function of time, it is possible to determine the project time-cost trade-off (TCTO) curve which provides the minimum possible cost of completing project in its feasible time range.

- **Literature review:-**

- **Lihui Zhang [1], Xin Zou [1], and Jianxun Qi [1] (2015)** - This paper presents a mixed integer nonlinear programming model that combines the general DTCTP and the concept of soft logic. The execution modes of an activity in different units are also considered. The DTCTP is known to be strongly NP-hard, and the introduction of soft logic makes it even more complex. A genetic algorithm (GA) is proposed to resolve the problem. The effectiveness of the proposed GA is verified using the example of a bridge construction project presented in the previous literature. The model proposed in this paper provides more flexibility to reduce the total cost and time of a repetitive project for the planners.

- **S. K. Biswasa, C. L. Karmakera, T. K. Biswasa (2016)** - The objective of the time-cost trade-off problem (TCTP) is to reduce the original project duration obtained from the critical path analysis, to meet a specific deadline with the minimum direct and indirect cost of the project. Direct costs include costs of material, labor, equipment etc. whereas indirect costs are the necessary costs of doing work which can‟t be related to a particular task. There are enormous research works in the arena of TCTP. In 1991, Shouman et al. constructed a framework using mixed integer linear programming and CPM and utilized in natural gas projects. The value of the study is that, by the use of it, minimum total cost is achieved using crashing concept. A survey on forty seven papers conducted by Agarwal et al. (2013) revealed that about 41% work was performed in construction area during the 1990-2002. Liu et al. (1995) developed a hybrid method using linear and integer programming for time-cost trade-off problem. Several researchers used dynamic programming to adjust between two important aspects of the project (Hindelang et al., 1979; Prabuddha et al., 1995; Arauzo et al., 2009).

- **Anagha Anirudh Galagali (2017)** - In today's competitive business environment, delivering projects in the least possible time, with maximum quality and minimum cost has got a critical issue for project managers. Project time crashing plays an important role in project management determining which activities duration to crash to complete the project in the stipulated time. It is suggested that the project quality may be affected by project crashing and an actual construction project has been considered to study the tradeoffs among time, cost, and quality. The purpose is to highlight the managerial insights gained, as well as pointing out key problems and difficulties faced.

- **Hongbo Li, Zhe Xu and Wenchao Wei (2018) -** Time/cost trade-offs in projects accelerate the execution of some activitiesby increasing the amount of non-renewable resources committed to them and therefore shortenthe project duration. The discrete time/cost trade-off problem (DTCTP) has been extensively studied during the past 20 years. However, due to its complexity, the DTCTP—especially the DTCTP curve problem (DTCTP-C)—has only been solved for relatively small instances. To the best of our knowledge, there is no computational performance analysis for solving the DTCTP-C on large project instances with up to 500 activities. The objective is to obtain a good appropriate efficient set for the large-scale instances. The first algorithm is based on the non-dominated sorting genetic algorithm II (NSGA-II) and uses a specially designed critical path-based crossover operator. The second algorithm is a steepest descent heuristic which generates efficient solutions by iteratively solving the DTCTP with different deadlines. Computational experiments are conducted to validate the proposed algorithms on a large set of randomly generated problem instances.

- **N.N. Klevanskiy*, S.I. Tkachev, L.A. Voloshchouk (2019)** - the projects as aggregations. Each aggregation is determined by solving a resource availability cost problem (RACP) model to complete the project by his critical path. In the second stage, aggregations are used for its resource leveling scheduling the multi-project by a pre-specified project duration. Both stages use priority rule (PR) heuristics. Both stages are based on two parallel schedule generation schemes (SGS) and resource criteria. Each SGS uses two PR. The first SGS, a set of demands must be developed as initial solution.
- **Reza Lotfi et.al (2020)** - This study aims to take into account the sustainability pillars in scheduling projects and uncertainties in modeling them. To model the study problem, robust nonlinear programming (NLP) involving the objectives of cost, quality, energy, and pollution level is applied with resource-constrained. According to the results, as time diminished, the cost, energy, and pollution initially decreased and then increased with a reduction in quality.
- **Abhilasha Panwar and Kumar Neeraj Jha (2021)** - Quality and safety are important and are the leading concerns in a construction project. Planning a project without properly incorporating these two performance parameters propagates multifarious defects and accidents in construction projects and this ultimately leads to cost overrun and time delay. In order to deal with quality and safety at the planning stage (along with time and cost), a decision-making model considering time, cost, quality, and safety (TCQS) is developed on the basis of a many-objective evolutionary algorithm (non dominated sorting genetic algorithm III) and presented in this study
- **Dang-Trinh Nguyen et.al (2022)** - In this paper, fuzzy logic is utilized to model the uncertainty embedding α-cut approach to see the effect of the uncertainty on the time, cost, and quality of the project. Then multi-objective Symbiotic Organism Search (SOS) algorithm is applied to find a set of the optimal solution in different uncertainty levels and provide the project manager several possible actions to implement the project. Two numerical case studies of a repetitive construction project were analyzed to see the effectiveness of the model and its capability to solve the TCQT problem in the construction project. The results showed that the proposed model is powerful to explore the solution for the shortest project duration with minimum incurred cost and high overall quality in the construction project. In comparison to the other widely used methods and other algorithms, the proposed model is proven to be effective and competitive in solving the TCQT problem.

- **Methodology and Examples :-**

Time Cost Trade- off Problems can be grounded on the number and order of coffers as well as on the type of relationship between duration and coffers as shown in Figure 1. Still, we face Time Resource Trade-off Problems (TRTP) where the duration of an exertion is assumed to be a non adding function of the quantum of the renewable resource, If a single renewable resource is considered similar as force or outfit. When this function is nonstop, we get Nonstop Time Resource Dicker Problems (CTRTP). Again, if this function is separate we gain Discrete Time Resource Trade-off Problems (DTRTP) in which any exertion can be executed according to several processing modes where a mode represents a possible allocation of the resource with a corresponding duration. As can be seen in Figure 1, quality has noway been integrated in Time Resource Trade-off Problems. Energy, plutocrat or raw accoutrements are exemplifications of single non renewable coffers. But in the literature, only plutocrat is actually considered as the single non renewable resource and corresponds to the cost of any order of coffers, which leads to Time Cost Trade-off Problems (TCTP). For case plutocrat as the non renewable resource can be the cost of overtime, which is a single renewable resource, or the cost of several renewable and non renewable coffers defined as the plutocrat spent for case on factors, accoutrements and force. In similar Time Cost Trade-off Problems, the cost of any exertion is a non adding function of its duration. This function can either be continous Time Cost Trade-off Problems, (CTCTP), or separate (Separate Time Cost Trade-off Problems, DTCTP). Three references introduce the quality in CTCTP (Babu and Suresh, 1996; Khang and Myint, 1999; Zhang etal., 2014). No lower than ten references consider the quality in DTCTP where each exertion can be reused in one mode amongst several prosecution modes, a mode corresponding to a combination of cost, duration and quality (Tran etal., 2015; Kim etal., 2012; Liberatore and Pollack-Jonhson, 2013; Tareghian and Taheri, 2006 and 2007; Mohammadipour and Sadjadi, 2016; El-Rayes and Kandil, 2005; Mungle etal., 2013; Monghasemi etal., 2015; Afshar etal., 2007.

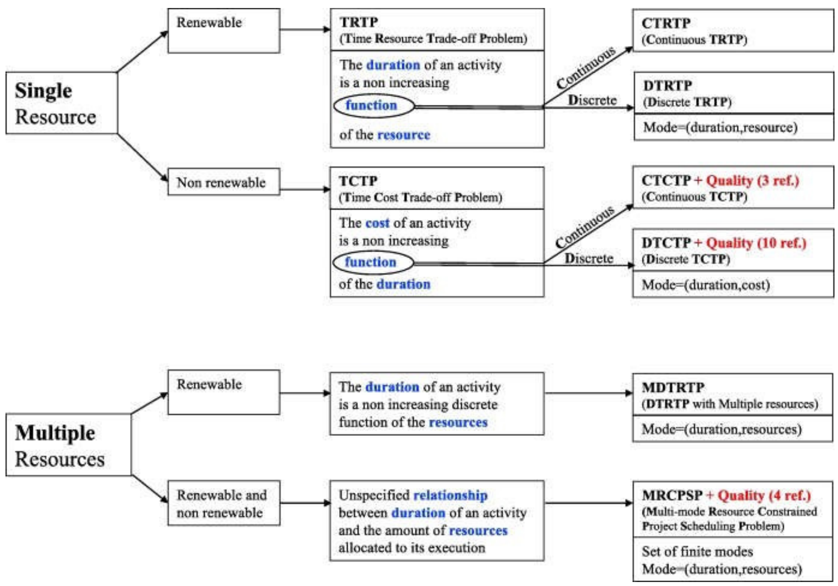


Figure 1: Classification of Time Resource or Cost Trade-off Problems with quality

Conversely we found four references dealing with quality in Multi mode Resource Constrained Project Scheduling Problems (MRCPSP) that include renewable and/or non renewable resources and for which the Resource Constrained Project Scheduling Problem (RCPSP) is a special case as there is a single execution mode (Icmeli-Tukel and Rom, 1997; Tiwari et al., 2009; Fu and Zhang, 2016; Afruzi et al., 2014). Several objectives, formulations and solution methods are considered in the literature, as shown in Figure 2. If the assessment of quality is binary, the objective is to minimize the rework cost or the rework duration. Otherwise, the objective is to minimize the project duration or its cost or to maximize the project quality as an aggregation of individual qualities (namely qualities of activities), the arithmetic mean being quite often used. Single objective formulations are most common and include one objective amongst the five aforementioned ones or make use of scalarisation approaches.

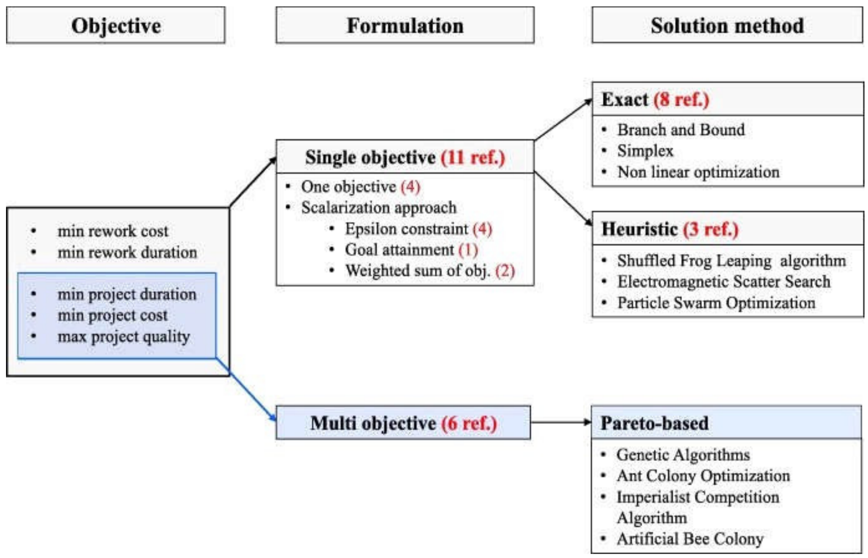


Figure 2: Formulations and solution methods

The Methodology adopted to crash the project to answer the Problem Statement consequently solving the Time-Cost-Quality Trade-off is depicted in the following points.

(I) **Using Microsoft Project to plan & schedule the project.**

- A myriad of details are considered in planning how to coordinate all the RCC activities, in developing a realistic schedule, Project Management software, Microsoft Project is the most commonly used software to deal with all the data needed to develop schedule information.
- The various activities are linked by the software in terms of their predecessors and successors.

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(II) Using Microsoft Excel to schedule the project with CPM

- Each activity is scheduled by calculating its earliest & latest times (ES/EF/LS/LF) in MS Excel with the help of specific formulae.
- The slack for an activity is the difference between its latest finish time and its earliest finish time. Thus knowing the earliest & latest times of each activity, their corresponding slack is calculated. Those activities with 0 slack will be classified as Critical activities.

(III) Using Microsoft Excel Solver to crash the project and solve the Time-Cost-Quality Trade off

- The problem of finding the least expensive way of crashing activities and the consequent Time-Cost-Quality Trade-off can be rephrased in a form more familiar to Microsoft Excel Sheet and solved using Microsoft Excel Solver Add-in.

(IV) Updating process of CPM

- During the process of implementation the plan according to the network, we may come across one or more the following possibilities:
- That some or all activities are progressing according to schedules;
- That some or all activities are ahead of schedule; and
- That some or all activity is behind schedule.

If all activities are progressing according to the schedule, there is no need for updating the network but this is seldom the case.

Therefore, based on the progress of the work and the revised durations of unfinished activities due to delays, the network diagram has to be redrawn and this process is known as updating**.**

When the project is partially completed and is at an intermediate stage, it may be possible that:

- The time duration originally assigned activities were erroneous and
- The planner may himself feel it desirable as a result of experience.

The process of replanning and rescheduling based on the result which serve a guidance for decision made by taking into consideration the new knowledge and latest information at an intermediate stage of the project thus modifying the original network, is known as the process of updating.

Illustrative example:

A building project consists of 10 activities, represented by the network shown below in figure. The normal distributions required to perform various activities of the above project are given in table below. Compute:

(a) Event time (b) Activity times and (c) Total Float. Also, determine the critical path.

Solution:

The updated network can now be drawn on the base of the first fig. the basis data of columns (1), (2), (4), (5) and (6) of the below table. For the activities which have already been completed.

This fig. shows original networks with T_E and T_L marked.

D

B E H

A I

C

G

Activity	Estimated Duration	Activity	Estimated Duration
A	5	F	2
B	2	G	3
C	6	H	8
D	4	I	7
E	4	J	2

Solution:

1. Computation of event times-

The earliest event times (T_E) and latest event occurrence times (T L) are computed in a tabular form, below in table.

Table

Computation of Event Times

Event No.	Earliest event time				Latest event time			
	Predecessor event (i)	t^{ij}	T^{j}_{E}	T_E	Successor event (j)	t^{ij}	T^{i}_{L}	T_L
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	---	-	-	0	2	5	0	0
2	1	5	5	5	3	2	6	
					4	6	5	5
3	2	2	7	7	5	4	9	
					6	4	8	8
4	2	6	11	11	5	2	11	
					7	3	15	11
5	3	4	11					
	4	2	13	13	8	7	13	13
6	3	4	11	11	8	8	12	12
7	4	3	14	14	8	2	18	18
	5	7	20					
8	6	8	19	20	-	-		20
	7	2	16					

TE =7 D TE =11

TL =8 4 TL =12

B E H

T_E=0 T_E=5 2 4 TE =13 8

A I TE =20

T_L=0 5 TL =5 C F TL =19 J TL=20

6 2 2

T_E=11 G TE =14

T_L=11 3 T_L=18

2. Computation of activity times and the total float-

The foremost start time (EST), foremost finish time (EFT), rearmost launch time (LST) and rearmost finish time (LFT) of each exertion, along with the total pier are reckoned in Table 1. The EST of each exertion is equal $T^i{}_E$, while EFT is equal to $T^i{}_E$ + tij. Also, LFT is equal to $T^i{}_L$ while LST is equal to $T^i{}_L$-tij, as explained before. Eventually, the total pier of each exertion is equal to either LST – EST or LFT – EFT.

Computation of Activity Time and Total Float

Activity (i – j)	Duration t^{ij}	Earliest		Latest		Total Float F_T
		Start Time (EST)	Finish Time (EFT)	Start Time (LST)	Finish Time (LFT)	
1	**2**	**3**	**4**	**5**	**6**	**7**
1-2	5	0	5	0	5	0
2-3	2	5	7	6	8	1
2-4	6	5	11	5	11	0
3-5	4	7	11	9	13	2
3-6	4	7	11	8	12	1
4-5	2	11	13	11	13	0
4-7	3	11	14	15	18	4
5-8	7	13	20	13	20	0
6-8	8	11	19	12	20	1
7-8	2	14	16	18	20	4

3. Location of critical path –

The activities for which total pier is zero, are the critical conditioning, and these are 1-2, 2 and 5-8. The critical path is thus along these conditioning, starting from event 1 and ending at event 8. The critical path is shown which thick lines in figure, along with the exertion time marked on each exertion.

TE =7 D (7;11) TE =11

TL =8 4(8; 12) TL =12

B(5;7) 2(6;8) E (7;11) (12;20) 8 H (11;19)

T_E=0 T_E=5 4(9;13) TE =13

A (0;5) I(13;20) TE =20

T_L=0 5(0;5) TL =5 C(5;11) F(11;13) TL =19 7 (13;20) TL=20

6 (5;11) 2(11;13) J (14;16) 2(18;20)

T_E=11 G(11;14) TE =14

T_L=11 3 (15; 18) T_L=18

CRITICAL PATH

Result:

As the result we can say the controlling is reciprocal to the planning. Once the schedule plan has been prepared and prosecution commenced control over the progress of work has to be exercised in order to complete the work by the quested date. Control involves comparing at regular intervals the factual achievement with the original plans and also taking any necessary corrective action to bring effects back on the schedule. Thus, Controlling needed an upward inflow of information through a suitably designed reporting system. The information so fed is anatomized and design plan is brought up to date with necessary variations to keep performance as per the schedule like we saw the last test there streamlined fig shows how we can complete the work of the design in lower days without detainments the condition.

Conclusion:-

The points of this exploration were time- cost trade-off analysis for a construction design. The analysis has been done for an being design critical path system (CPM) and a heuristic system have been used to find out the cash time and crash costs the Retrogression analysis has been done in order to developed the relationship between the crash times and cash costs the relation between crash time and crash costs has led to developed the optimization model. Mathematical program has been used to get the minimal total costs of the design with the minimal durations. All these ways employed in this paper have shown satisfactory results.

References:-

1. Demeulemeester, E.L.; Herroelen, W.S. Project Scheduling: A Research Handbook; Kluwer Academic Pub: Dordrecht, The Netherlands, 2002.
2. Dobrovolskien˙ e, N.; Tamoši¯ unien˙ e, R. Sustainability-oriented financial resource allocation in a project portfolio through multi-criteria decision-making. Sustainability 2016, 8, 485. [CrossRef]
3. Li, H.; Dong, X. Multi-mode resource leveling in projects with mode-dependent generalized precedence relations. Expert Syst. Appl. 2018, 97, 193–204. [CrossRef]
4. Li, H.; Xiong, L.; Liu, Y.; Li, H. An effective genetic algorithm for the resource levelling problem with generalized precedence relations. Int. J. Prod. Res. 2018, 56, 2054–2075. [CrossRef]
5. Harvey, R.T.; Patterson, J.H. An implicit enumeration algorithm for the time/cost tradeoff problem in project network analysis. Found. Control Eng. 1979, 4, 107–117.
6. Hindelang, T.J.; Muth, J.F. A dynamic programming algorithm for decision CPM networks. Oper. Res. 1979, 27, 225–241. [CrossRef]
7. Brucker, P.; Drexl, A.; Möhring, R.; Neumann, K.; Pesch, E. Resource-constrained project scheduling: Notation, classification, models, and methods. Eur. J. Oper. Res. 1999, 112, 3–41. [CrossRef].
8. Ahuja, R.K., Magnati, T.L., Orlin, J.B., Network Flows:Theory, Algorithms and Apllications , Prentice Hall, Englewood Cliffs, NJ, 1993, pp.164 - 165.
9. Bellman, R., On a Routing Problem, Quarterly of Applied Mathematics, 16(1), 1958, pp. 87 - 90.
10. Csordas, H., Malyusz, L., A Network Flow Algorithm For Time-Cost Trade-off With Technological Decision, 7th International Conference Organization, Technology and Management in Construction, Zadar, Croatia, 2006
11. Csordas, H., Optimal Selection of Recourses in Projects Based on the Classical Time - Cost Trade – Offs , Hungary, Periodica Polytechnica Social and Management Scienses, 2009, Vol. 17, No. 1, pp. 47 - 55.
12. Csordas, H., Activities With Multi-Parameters In Time-Cost Trade-Off, Hungary, Pollack Periodica, 2011, Vol. 6, No. 2, pp. 37 – 48.
13. Dijkstra, E. W., A Note on Two Problems in Connexion With Graphs, Numerische Mathematik, Vol.1., 1959, pp. 269 - 271.
14. Franck, B., Neumann, K., Schwindt, C., Project scheduling with calendars OR Spektrum, Vol 23., 2001, pp. 325-334.
15. Fulkerson, R. D., A network flow computation for project cost curves, Management Science Vol. 2. No. 2. January, 1961, pp. 167 - 168.
16. Hajdu M., Malyusz L., How would you like it: cheaper or shorter?, Organization, Technology & Management in Construction: An International Journal, Vol. 1., No. 2., 2009, pp. 59 – 63.
17. Hajdu M., Klafszky E., An algorithm to solve the cost optimization problem through an activity on arrow type network (CPM/cost problem), Periodica Polytechnica ser. Architecture, Vol. 37., Nos 1-4., 1993, pp. 27 - 40.

18. Kelley, J. E., Critical Path Planning and Scheduling: Mathematical Basis, Operation Research, Vol. 9. No. 3., 1959
19. Kelley, J. E. Walker, M. R., Critical Path Planning and Scheduling, Proc. the Eastern Joint Computer Conference, Boston, 1959
20. Klafszky E., Hálózati folyamok, Budapest, 1969.
21. W. Crowston and G. L. Thompson, "Decision CPM: A method for simultaneous planning, scheduling, and control of projects," Oper. Res., vol. 15, pp. 407–426, 1967.
22. A. Aghaie and H. Mokhtari, "Ant colony optimization algorithm for stochastic project crashing problem in PERT networks using MC simulation," Int. J. Adv. Manuf. Technol., vol. 45, pp. 1051–1067, 2009
23. D. R. Fulkerson, "A network flow computation for project cost curves,"Manage. Sci., vol. 7, pp. 167–178, 1961
24. P. S. Pulat and S. J. Horn, "Time-resource tradeoff problem," IEEE Trans. Eng. Manage., vol. 43, no. 4, pp. 411–417, Nov. 1996.
25. E. B. Berman, "Resource allocation in PERT network under activity continuous time-cost functions," Manage. Sci., vol. 10, pp. 734–745, 1964.
26. P. Vrat and C. Kriengkrairut, "A goal programming model for project crashing with piecewise linear time-cost trade-off," Eng. Costs Prod. Econ., vol. 10, pp. 161–172, 1986.
27. J. R. Kelley, "Critical-path planning and scheduling: Mathematical basis," Oper. Res., vol. 9, pp. 296–320, 1961.
28. R. F. Deckro, J. E. Hebert, W. A. Verdini, P. H. Grimsurd, and E. Venkateshwar, "Nonlinear time-cost tradeoff models in project management," Comput. Ind. Eng., vol. 28, pp. 219–229, 1995.
29. S. A. Burns, L. Liu, and C. Weifeng, "The Lp/Ip hybrid method for construction time cost trade of analysis," Constr. Manage. Econ., vol. 14, pp. 265–275, 1996.