Kronecker Product of Even Cycles with Some Transformation Graphs

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Abstract:

Domination is an interesting research area in graph theory. Various domination parameters have been dicussed by many authors. In this paper, we have found the general formula for the Kronecker product of cycles of even length with some of its transformation graphs.

Keywords: Bipartite, Cycle, Domination number, Kronecker product graph, transformation graph.

I. INTRODUCTION

Graph theory is one of the florescent area to find the solution for some unsolved problems in real life which are motivated by objects and relation between them. A graph $G = (V, E)$, where V is a finite set of elements called vertices and E is a set of unordered pairs of distinct vertices of G called edges. The degree of a vertex v in G is the number of edges incident on it.

II. PRELIMINARIES

Definition: 2.1

A graph G is said to be bipartite if the vertex set of $V(G)$ can be partitioned in to two subsets Xand Y such that every edge of G has one end in X and the other end in Y. A bipartite graph G with $|\mathrm{X}| = m$ and $|Y| = k$ is said to be complete if every element in one partition is adjacent with all elements of the other partition and is denoted by $K_{m,n}$. The graph $K_{1,n}$ is called a star graph.

Definition: 2.2

A set $D \subseteq V$ is a dominating set of G if every vertex $v \in V - D$ is adjacent to at least one vertex of D; A dominating set D is minimum if there is no dominating set D' with $|D'| < |D|$. The cardinality of a minimum dominating set is called the domination number denoted by $\gamma(G)$ and the minimum dominating set D of G is also called a $\gamma -$ set.

Definition:2.3

If $G_1$and G_2 are two graphs with vertex sets V_1 and V_2 respectively then their product graph is a graph denoted by $G_1(K)G_2$ with its vertex set as $V_1 \times V_2$ where u_1v_1 is adjacent with u_2v_2 if and only if $u_1u_2 \in E_1$and $v_1v_2 \in E_2$is called the **Kronecker product** of graphs.

Definition:2.4

Let $G = (V(G), E(G))$ be a graph and x, y, z be three variables taking values + or -. The **transformation graph** G^{xyz} is the graph having $V(G) \cup E(G)$ as the vertex set and for $\propto, \beta \in V(G) \cup E(G)$, $\propto akd\ \beta$ are adjacent in G^{SyZ} if and only if one of the following holds:

(i) $\propto, \beta \in V(G)$. $\propto akd\ \beta$ are adjacent in G if x = +; $\propto akd\ \beta$ are not adjacent in G if x = - .

(ii) $\propto, \beta \in E(G)$. $\propto akd\ \beta$ are adjacent in G if y = +; $\propto akd\ \beta$ are not adjacent in G if y = - .

(iii)$\propto \in V(G), \beta \in E(G)$. $\propto akd\ \beta$ are incidentt in G if z = +; $\propto akd\ \beta$ are not incident in G if z = - .

III. MAIN RESULTS

Result :3.1

Let $G = C_n$ be a cycle of order n and G^{SyZ} be a transformation of G with $2k$ vertices . Then

(i) $\gamma(G^{+++}) =]\frac{2n}{5}]$

(ii) $\gamma(G^{++-}) = 2$

(iii) $\gamma(G^{+-+}) = \begin{cases} 2 & \text{if } k = 3,4 \\ ]\frac{n}{3}] + 2 & \text{if } k = 6,9,12,\dots \\ ]\frac{n}{3}] + 1 & otherwise \end{cases}$

(iv) $\gamma(G^{-++}) = \begin{cases} 2 & \text{if } k = 3 \\ ]\frac{n}{3}] + 1 & \text{if } k > 3 \end{cases}$

(v) $\gamma(G^{+--}) = 3$

(vi) $\gamma(G^{-+-}) = \begin{cases} 2 & \text{if } k = 4\,,5 \\ 3 & \text{if } k = 3\ akd\ k > 5 \end{cases}$

(vii) $\gamma(G^{--+}) = \begin{cases} 2 & \text{if } k = 3 \\ 3 & \text{if } k > 3 \end{cases}$

(viii) $\gamma(G^{---}) = \begin{cases} 3 & \text{if } k = 3\ akd\ 4 \\ 2 & \text{if } k > 4 \end{cases}$

Theorem : 3.2

Let G be a cycle of order n which is even and $k > 4$ and G^{++-} be the transformation graph of G. Then $\gamma(G\ (K)G^{++-}) = 6\ ]\frac{n}{4}]$.

Proof :

Let $G = C_n$, $k > 4$ be a even order cycle .
Let G^{++-} be the transformation of G and $(G^{++-}) = \{u_j, e_j / \ 1 \leq j \leq m\}$ $|V(G^{++-})| = 2m$.
Let us denote $G^* = G(K)G^{++-}$.
$|V(G^*)| = 2mk$.
The adjacency in $V(G^*)$ is as follows:

$$N(u_iv_1) = \{u_kv_2, u_kv_m\} \cup \{u_ke_j / \ 2 \leq j \leq m-1\}$$
$$N(u_ie_1) = \{u_ke_2, u_ke_m\} \cup \{u_kv_j / \ 3 \leq j \leq m\}$$
$$N(u_iv_m) = \{u_kv_1, u_kv_{m-1}\} \cup \{u_ke_j / \ 1 \leq j \leq m-2\}$$
$$N(u_ie_m) = \{u_ke_1, u_ke_{m-1}\} \cup \{u_kv_j / \ 2 \leq j \leq m-1\} ,$$
$$N(u_iv_j) = \{u_kv_{j-1}, u_kv_{j+1}\} \cup \{u_ke_h / \ 1 \leq h \leq m , h \neq j-1, j\}$$
$$N(u_ie_j) = \{u_ke_{j-1}, u_ke_{j+1}\} \cup \{u_kv_h / \ 1 \leq h \leq m , h \neq j, j+1\}$$

if $= 1$, $k = 2, k$ and if $i = k$, $k = 1, k-1$
and for all $2 \leq i \leq k-1$, $k = i-1, i+1$ and $1 \leq j \leq m$.

Therefore, $d(u_iv_j) = d(u_ie_j) = 2k$ for all $1 \leq i \leq k$, $1 \leq j \leq m$.
Let D be a dominationg set for G^*.
$D = (a, b) / \ a \subset S_1, b \subset S_2$, where S_1 and S_2 such that

$$S_1 = \{\{u_Sv_j, u_Sv_{j+1}, u_Se_{j-1}\}, \{u_Sv_j, u_Sv_{j+1}, u_Se_{j+1}\}, \{u_Se_j, u_Se_{j+1}, u_Sv_j\}, \{u_Se_j, u_Se_{j+1}, u_Sv_{j+2}\}, \{u_Sv_j, u_Sv_{j+3}, u_Se_{j+1}\}, \{u_Se_j, u_Se_{j+3}, u_Sv_{j+2}\}, \{u_Sv_j, u_Sv_{m-2}, u_Se_{m-1}\}, \{u_Se_j, u_Se_{m-2}, u_Sv_m\}\}$$

$$S_2 = \{\{u_yv_j, u_yv_{j+1}, u_ye_{j-1}\}, \{u_yv_j, u_yv_{j+1}, u_ye_{j+1}\}, \{u_ye_j, u_ye_{j+1}, u_yv_j\}, \{u_ye_j, u_ye_{j+1}, u_yv_{j+2}\}, \{u_yv_j, u_yv_{j+3}, u_ye_{j+1}\}, \{u_ye_j, u_ye_{j+3}, u_yv_{j+2}\}, \{u_yv_j, u_yv_{m-2}, u_ye_{m-1}\}, \{u_ye_j, u_ye_{m-2}, u_yv_m\}\}$$

for all $2 \leq j \leq m-1, x = 2p-1, y = 2p$, $p = \begin{cases} 1,3,5, \ldots, \frac{n}{2}, & \text{if } \frac{n}{2} \text{ is odd} \\ 1,3,5, \ldots, \frac{n-2}{2}, & \text{if } \frac{n}{2} \text{ is even} \end{cases}$

By the selection of S^* and S^{**} , each suffix x dominate exactly two partitions.
We have $2mk$ vertices in G^* and degree of each vertex is $2k$.

For, $N(u_iv_j) \cup N(u_iv_{j+1}) = \{u_kv_{j-1}, u_kv_j, u_kv_{j+1}, u_kv_{j+2}\} \cup \{u_ke_1, u_ke_2, \ldots, u_ke_{j-1}, u_ke_{j+1}, \ldots, u_ke_{m-1}, u_ke_m\}$

$N(u_ie_{j-1}) = \{u_kv_1, u_kv_2, \ldots, u_kv_{j-2}, u_kv_{j+1}, \ldots, u_kv_m\} \cup \{u_ke_{j-2}, u_ke_j\}$

$N(u_ie_{j+1}) = \{u_kv_1, u_kv_2, \ldots, u_kv_j, u_kv_{j+3}, \ldots, u_kv_m\} \cup \{u_ke_j, u_ke_{j+2}\}$

if $i = 1$, $k = 2, k$ and if $i = k$, $k = 1, k-1$
and for all $2 \leq i \leq k-1$, $k = i-1, i+1$ and $2 \leq j \leq m-1$.

Since $\{u_iv_j, u_iv_{j+1}\}$ dominate $8 + 2(m-1)$ vertices , it is must to choose one more vertex .

Hence, $N(u_i v_j) \cup N(u_i v_{j+1}) \cup N(u_i e_{j-1}) = \{u_k v_j, u_k e_j / k = i-1, i+1\}$

$N(u_i v_j) \cup N(u_i v_{j+1}) \cup N(u_i e_{j+1}) = \{u_k v_j, u_k e_j / k = i-1, i+1\}$,

$1 \le j \le m$ for a fixed i, $2 \le i \le k-1$

and if $i = 1$, $k = 2, k$ and if $i = k$ then $k = 1, k-1$.

$\Longrightarrow \{u_i v_j, u_i v_{j+1}, u_i e_{j+1}\}$ for all i and j , it is one of the minimum dominating set for G^*.

$\Longrightarrow D$ is the minimum dominating set and the domination number

$\gamma(G^*) = 2\,(3\,]\frac{n}{4}])$.

Theorem : 3.3

Let G be the Kronecker product of G_1 and G_2 where $G_1 = C_n$, k is even, $k > 5$ and $G_2 = G_1^{+-+}$. Then $\gamma(G) = k\,]\frac{n}{4}]$.

Proof :

Let $G_1 = C_n$, $k > 5$ be a graph of order even and $G_2 = G_1^{+-+}$ be the transformation graph of G_1.

Let us denote $V(G_1) = \{ u_i / 1 \le i \le k \}$

$V(G_2) = \{ v_j, e_j / 1 \le j \le k \}$

In G_2 , the adjacency of v_j is $\{ v_{j-1}, v_{j+1}, e_{j-1}, e_{j+1} \}$, $1 \le j \le k$.

Since the variable ‘y’ is ‘-’ , each e_j is adjacent with all $e_k / 1 \le k \le k$ except e_{j-1} and e_{j+1}; and also adjacent with v_j and v_{j+1} .

Let $G = G_1(K)G_2$, $V(G) = \{u_i v_j, u_i e_j / 1 \le i \le k , 1 \le j \le k \}$; $|V(G)| = 2k^2$.

$N(u_i e_j) = \{\{u_{i-1} e_k / 1 \le k \le k , k \ne j-1, j\, akd\, j+1\} \cup$

$\{u_{i+1} e_k / 1 \le k \le k, k \ne j-1, j\, akd\, j+1 \} \cup$

$\{u_{i-1} v_j, u_{i-1} v_{j+1}, u_{i+1} v_j, u_{i+1} v_{j+1}\} \}$, $1 \le i, j \le k$.

$N(u_i v_j) = \{u_{i-1} v_h , u_{i+1} v_h , u_{i-1} e_k , u_{i+1} e_k / h = j-1, j+1; k = j-1, j\}$,

for all $1 \le i, j \le k$.

Hence, $d(u_i e_j) = 2(k-1)$, $d(u_i v_j) = 8$, for all $1 \le i, j \le k$.

In G, there are S^* and S^{**} having $\frac{n}{2}$ sub partitions and each sub partition consists $2k$ elements. Hence $|S^*| = |S^{**}| = k^2$.

Also, elements from one sub partition of S^* dominate exactly two sub partitions of S^{**}.

Therefore, vertices from $\frac{n/2}{2}$ sub partitions of S^* dominate all elements of S^{**}.

By the adjacency of $e_j \subset V(G_2)$, the set of vertices $\{e_j / j \text{ is } odd , 1 \le j \le k \}$ dominate all $\{v_j / 1 \le j \le k\}$ independently.

Similarly, $\{e_j / j \text{ is } evek, 1 \le j \le k \}$ dominate all $\{v_j / 1 \le j \le k \}$ independently.

Also any pair of vertices $\{e_j, e_k\} \subset V(G_2)$, $k \neq j-2, j-1, j+1$ and $j+2$, $1 \leq j \leq k$ dominate all $\{e_j / 1 \leq j \leq k\}$.

Let us choose $D_1 \subseteq S^*$ and $D_2 \subseteq S^{**}$

$$D_1 = \{\{u_{2i-1}e_1, u_{2i-1}e_3, u_{2i-1}e_5, \dots, u_{2i-1}e_{n-1}\}, \{u_{2i-1}e_2, u_{2i-1}e_4, u_{2i-1}e_6, \dots, u_{2i-1}e_n\}\}$$

$$D_2 = \{\{u_{2i}e_1, u_{2i}e_3, u_{2i}e_5, \dots, u_{2i}e_{n-1}\}, \{u_{2i}e_2, u_{2i}e_4, u_{2i}e_6, \dots, u_{2i}e_n\}\}$$

$$i = \begin{cases} 1,3,5 \dots, \frac{n-2}{2} & \text{if } \frac{n}{2} \text{ is } evek \\ 1,3,5, \dots, \frac{n}{2} & \text{if } \frac{n}{2} \text{ is } odd \end{cases}$$

Let $x \in D_1, y \in D_2$.

$N(x) = S^{**}$ and $N(y) = S^*$.

Hence, $D = \{(x,y) / x \in D_1, y \in D_2\}$ is a dominating set.

x consists of $]\frac{n}{4}] \binom{n}{2}$ elements and similarly y consists of $]\frac{n}{4}] \binom{n}{2}$ elements.

Since each element of D dominate all $u_i v_j$ independently, D is the required minimum dominating set for G.

$$\Longrightarrow \gamma(G) = 2\,]\frac{n}{4}]\binom{n}{2} = k\,]\frac{n}{4}].$$

Result : 3.4

In the above theorem, if $\frac{n}{2}$ is odd ,

$$N(u_1e_j) = \{u_2v_j, u_2e_j, u_nv_j, u_ne_j\};$$
$$N(u_{n-1}e_j) = \{u_{n-2}v_j, u_{n-2}e_j, u_nv_j, u_ne_j\}$$
$$N(u_2e_j) = \{u_1v_j, u_1e_j, u_3v_j, u_3e_j\};$$
$$N(u_ne_j) = \{u_1v_j, u_1e_j, u_{n-1}v_j, u_{n-1}e_j\}, 1 \leq j \leq k.$$

$\{u_1e_j, u_{n-1}e_j\} \in S^*$ dominate $\{u_nv_j, u_ne_j\} \in S^{**}$ twice.

$\{u_2e_j, u_ne_j\} \in S^{**}$ dominate $\{u_nv_j, u_ne_j\} \in S^*$ twice.

Therefore, D does not dominate the sub partitions independently.

Result : 3.5

If $G = C_n, k$ is even, then G^{-++} and G^{+-+} are isomorphic.

$\gamma(G^{-++}) = \gamma(G^{+-+}) = \frac{k}{2}$.

By the previous theorem, $\gamma(G(K)G^{-++}) = \gamma(G(K)G^{+-+}) = k\,]\frac{n}{4}]$.

Theorem: 3.6

Let $G = G_1(K)G_1{}^{+--}$ be the graph, $G_1 = C_n$, $k > 5$, k is even. Then $\gamma(G) = 4\,]\frac{n}{4}]$.

Proof :

Let $G = G_1(K)G_2$ where $G_1 = C_n$, $k > 5$, k is even and $G_2 = G_1{}^{+--}$.
$V(G_1) = \{ u_i / 1 \leq i \leq k \}$
$V(G_2) = \{ v_j, e_j / 1 \leq j \leq k \}$
$V(G) = \{u_i v_j, u_i e_j / 1 \leq i \leq k , 1 \leq j \leq k \}$
The adjacency of $V(G)$ as follows :
$N(u_i v_j) = \{u_{i-1}v_{j-1}, u_{i-1}v_{j+1}, u_{i+1}v_{j-1}, u_{i+1}v_{j+1}\} \cup$
$\{u_{i-1}e_k, u_{i+1}e_k / 1 \leq k \leq k , k \neq j-1 \; akd \; j \}, 1 \leq i, j \leq k.$
$N(u_i e_j) = \{u_{i-1}v_k, u_{i+1}v_k / 1 \leq k \leq k , k \neq j \; akd \; j+1\} \cup$
$\{u_{i-1}e_k, u_{i+1}e_k / 1 \leq k \leq k , k \neq j-1, j \; akd \; j+1 \}, 1 \leq i, j \leq k.$
$d(u_i v_j) = 2k$; $d(u_i e_j) = 4k - 10$
It is clear that $d(u_i v_j) < d(u_i e_j)$ for all $1 \leq i, j \leq k$.
Choose D_1 and D_2 such a way that,

$$D_1 = \{u_{2i-1}e_j, u_{2i-1}e_k , u_{2i}e_j, u_{2i}e_k\}$$

$$D_2 = \{u_{2i+1}e_j, u_{2i+1}e_k , u_{2i+2}e_j, u_{2i+2}e_k\} , i = \begin{cases} 1,3,5,\dots,\frac{n-2}{2} & \text{if } \frac{n}{2} is \; evek \\ 1,3,5,\dots,\frac{n}{2} & \text{if } \frac{n}{2} is \; odd \end{cases},$$

for all $1 \leq j, k \leq k$, $k \neq j-2, j-1, j+1 \; akd \; j+2$.
We know that, $|V(G)| = 2k^2$, $|S^*| = |S^{**}| = k^2$.
For all akd j, $d(u_i e_j) = 4k - 10$.
S^* and S^{**} have $\frac{n}{2}$ sub partitions and each sub partition having $2k$ elements.
Elements of each sub partition of S^* exactly dominate the elements in two sub partitions of S^{**}.
Choose a pair of vertices $(u_i e_j, u_i e_k)$ such that $k \neq j-2, j-1, j+1 \; akd \; j+2$; $1 \leq j \leq k$.
Then , $N(u_i e_j) \cup N(u_i e_k) = \{u_{i-1}v_j, u_{i-1}e_j, u_{i+1}v_j, u_{i+1}e_j\} / 1 \leq j, k \leq k.$
$|N(u_i e_j) \cap N(u_i e_k)| = 2(2k - 10).$
$|N(u_i e_j) \cup N(u_i e_k)| = |N(u_i e_j)| + |N(u_i e_k)| - |N(u_i e_j) \cap N(u_i e_k)| = 4k$
Therefore , $]\frac{n}{4}]$ number of pair of selected vertices from S^* dominate S^{**}.
In similar, $]\frac{n}{4}]$ number of pair of selected vertices $u_i e_j \in S^{**}$ dominate S^*.
Hence, D_1 and D_2 are the required minimum dominating sets with cardinality $4\,]\frac{n}{4}]$.
Hence $\gamma(G) = 4\,]\frac{n}{4}]$.

Corollary:3.7

If $\frac{n}{2}$ is odd then for all $\mathrm{i} = \begin{cases} 1,3,5, \dots, \frac{k-2}{k^2} & \text{if } \frac{k}{2} \text{ is } evek \\ 1,3,5, \dots, \frac{k}{2} & \text{if } \frac{k}{2} \text{ is } odd \end{cases}$,

and $1 \leq \mathrm{j}, k \leq k,\ k \neq \mathrm{j}-2, \mathrm{j}-1, \mathrm{j}+1\ akd\ \mathrm{j}+2$,

$N(D_1) = N(u_{2\mathrm{i}-1}\mathrm{e_j}) \cup N(u_{2\mathrm{i}-1}\mathrm{e_k}) \cup N(u_{2\mathrm{i}}\mathrm{e_j}) \cup N(u_{2\mathrm{i}}\mathrm{e_k}) = V(G)$.

By the adjacency mentioned in Result : 3.4, the graph G in above theorem : 3.6 with all i, j, k,

$$|N(u_{2\mathrm{i}-1}\mathrm{e_j})| + |N(u_{2\mathrm{i}-1}\mathrm{e_k})| = 2\left(\left]\frac{k}{4}\right[(4k-10) - (2k-5)\right)$$

$$|N(u_{2\mathrm{i}-1}\mathrm{e_j}) \cap N(u_{2\mathrm{i}-1}\mathrm{e_k})| = \frac{k}{2}(2k-10)$$

Hence, $N(D_1) = |N(u_{2\mathrm{i}-1}\mathrm{e_j}) \cup N(u_{2\mathrm{i}-1}\mathrm{e_k}) \cup N(u_{2\mathrm{i}}\mathrm{e_j}) \cup N(u_{2\mathrm{i}}\mathrm{e_k})|$

$$= |N(u_{2\mathrm{i}-1}\mathrm{e_j})| + |N(u_{2\mathrm{i}-1}\mathrm{e_k})| + |N(u_{2\mathrm{i}}\mathrm{e_j})| + |N(u_{2\mathrm{i}}\mathrm{e_k})|$$
$$-|N(u_{2\mathrm{i}-1}\mathrm{e_j}) \cap N(u_{2\mathrm{i}-1}\mathrm{e_k})| - |N(u_{2\mathrm{i}}\mathrm{e_j}) \cap N(u_{2\mathrm{i}}\mathrm{e_k})|$$
$$= 4\left(\left]\frac{n}{4}\right](4k-10) - (2k-5)\right) - 2\left(\frac{n}{2}(2k-10)\right)$$
$$= 2k^2 = |V(G)|.$$

Hence for both either $\frac{n}{2}$ is odd or $\frac{n}{2}$ is even, $\gamma(G) = 4\left]\frac{n}{4}\right]$.

Corollary : 3.8

For any cycle $G = C_n$, $\gamma(G^{+--}) = \gamma(G^{-+-}) = 2$.

Clearly, G^{+--} is isomorphic to G^{-+-} .

Therefore, for any cycle $G_1 = C_n$, k is even, $G = G_1(K)G_2$ is isomorphic to $G = G_1(K)G_3$, where $G_2 = G^{-+-}$ and $G_3 = G^{+--}$.

$\Longrightarrow \gamma(G) = 4\left]\frac{n}{4}\right]$.

Theorem :3.9

Let G^* be the Kronecker product of G and G^{--+} where $G = C_n$, k is even and $k > 5$. Then $\gamma(G^*) = 8\left]\frac{n}{4}\right]$.

Proof:

Let $G^* = G(K)G^{--+}$, G is an even order cycle, $k > 5$.

$|V(G)| = k$; $|V(G^{--+})| = 2k$; $|V(G^*)| = 2k^2$.

The neighbors of $V(G^*)$ as follows:

$$N(u_\mathrm{i}v_\mathrm{j}) = \{u_{\mathrm{i}-1}\mathrm{e}_{\mathrm{j}-1}, u_{\mathrm{i}-1}\mathrm{e_j}, u_{\mathrm{i}+1}\mathrm{e}_{\mathrm{j}-1}, u_{\mathrm{i}+1}\mathrm{e_j}\} \cup$$
$$\{u_{\mathrm{i}-1}v_\mathrm{k}, u_{\mathrm{i}+1}v_\mathrm{k} /\ 1 \leq k \leq k\ , k \neq \mathrm{j}-1, \mathrm{j}\ akd\ \mathrm{j}+1\ \},\ 1 \leq \mathrm{i}, \mathrm{j} \leq k.$$

$$N(u_\mathrm{i}\mathrm{e_j}) = \{u_{\mathrm{i}-1}v_\mathrm{j}, u_{\mathrm{i}-1}v_{\mathrm{j}+1}, u_{\mathrm{i}+1}v_\mathrm{j}, u_{\mathrm{i}+1}v_{\mathrm{j}+1}, u_{\mathrm{i}-1}\mathrm{e_k}, u_{\mathrm{i}+1}\mathrm{e_k}/$$

$1 \leq k \leq k, k \neq \mathrm{j}-1, \mathrm{j}\ akd\ \mathrm{j}+1 \}, 1 \leq \mathrm{i}, \mathrm{j} \leq k.$

For all i and , $d(u_{\mathrm{i}} v_{\mathrm{j}}) = 2k-2$; $d(u_{\mathrm{i}} \mathrm{e}_{\mathrm{j}}) = 2k-2$.

From the adjacency of $V(G^{*})$, it is clear that each $u_{\mathrm{i}} v_{\mathrm{j}} \in S^{*}$ is adjacent with $2(k-3)$ number of $u_h v_{\mathrm{k}} \in S^{**}$ and four number of $u_h \mathrm{e}_{\mathrm{k}} \in S^{**}$;

Similarly , each $u_{\mathrm{i}} \mathrm{e}_{\mathrm{j}} \in S^{*}$ is adjacent with $2(k-3)$ number of $u_h \mathrm{e}_{\mathrm{k}} \in S^{**}$ and four number of $u_h v_{\mathrm{k}} \in S^{**}$, for all $1 \leq \mathrm{i}, \mathrm{j}, k \leq k$; $k \neq \mathrm{j}-1, \mathrm{j}\ akd\ \mathrm{j}+1$.

Let us choose $D_1 \subseteq S^{*}$ and $D_2 \subseteq S^{**}$ such that

$$D_1 = \{\{u_{2\mathrm{i}-1} v_{\mathrm{j}}, u_{2\mathrm{i}-1} v_{\mathrm{k}}, u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{j}}, u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{k}}\}, \{u_{2\mathrm{i}+1} v_{\mathrm{j}}, u_{2\mathrm{i}+1} v_{\mathrm{k}}, u_{2\mathrm{i}+1} \mathrm{e}_{\mathrm{j}}, u_{2\mathrm{i}+1} \mathrm{e}_{\mathrm{k}}\}\}$$

$$D_2 = \{\{u_{2\mathrm{i}} v_{\mathrm{j}}, u_{2\mathrm{i}} v_{\mathrm{k}}, u_{2\mathrm{i}} \mathrm{e}_{\mathrm{j}}, u_{2\mathrm{i}} \mathrm{e}_{\mathrm{k}}\}, \{u_{2\mathrm{i}+2} v_{\mathrm{j}}, u_{2\mathrm{i}+2} v_{\mathrm{k}}, u_{2\mathrm{i}+2} \mathrm{e}_{\mathrm{j}}, u_{2\mathrm{i}+2} \mathrm{e}_{\mathrm{k}}\}\}$$

$$\text{for all i} = \begin{cases} 1,3,5,\ldots,\frac{n-2}{2} & \text{if } \frac{n}{2} \text{ is } evek \\ 1,3,5,\ldots,\frac{n}{2} & \text{if } \frac{n}{2} \text{ is odd} \end{cases}, 1 \leq \mathrm{j}, k \leq k, k \neq \mathrm{j}-2, \mathrm{j}-1, \mathrm{j}+1\ akd\ \mathrm{j}+2$$

Choose the pairs $(u_{2\mathrm{i}-1} v_{\mathrm{j}}, u_{2\mathrm{i}-1} v_{\mathrm{k}})$ and $(u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{j}}, u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{k}})$ such that $1 \leq \mathrm{j}, k \leq k,\ k \neq \mathrm{j}-2, \mathrm{j}-1, \mathrm{j}+1\ akd\ \mathrm{j}+2$.

Every pair $(u_{2\mathrm{i}-1} v_{\mathrm{j}}, u_{2\mathrm{i}-1} v_{\mathrm{k}}) \in S^{*}$ dominate $2(2k-6)$ vertices of S^{**} and $|N(u_{2\mathrm{i}-1} v_{\mathrm{j}}) \cap N(u_{2\mathrm{i}-1} v_{\mathrm{k}})| = 2k-12$.

Similarly,every pair $(u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{j}}, u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{k}}) \in S^{*}$ dominate $2(2k-6)$ vertices of S^{**} and $|N(u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{j}}) \cap N(u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{k}})| = 2k-12$.

Case : (i) If $\frac{n}{2}$ is even

Then the elements of each sub partition of S^{*} is adjacent with the elements of exactly two sub partitions of S^{**}. S^{*} and S^{**} have $\frac{n}{2}$ sub partitions.

Each sub partition having $2k$ elements.

$$|N(u_{2\mathrm{i}-1} v_{\mathrm{j}})| + |N(u_{2\mathrm{i}-1} v_{\mathrm{k}})| + |N(u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{j}})| + |N(u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{k}})|$$
$$-[|N(u_{2\mathrm{i}-1} v_{\mathrm{j}}) \cap N(u_{2\mathrm{i}-1} v_{\mathrm{k}})| + |N(u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{j}}) \cap N(u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{k}})|]$$
$$= 4(2k-6) - 2(2k-12) = 4k$$

which is the total number of elements in two sub partitions.

So we need minimum $2 \left]\frac{n}{4}\right]$ pair of vertices from S^{*} to dominate all the vertices of S^{**}.

In similar, $4 \left]\frac{n}{4}\right]$ vertices from S^{**} dominate all the vertices of S^{*}.

Case : (ii) If $\frac{n}{2}$ is odd

By the adjacency shown in Result : 3.4 , vertices $\{u_{2\mathrm{i}-1} v_{\mathrm{j}}, u_{2\mathrm{i}-1} v_{\mathrm{k}}, u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{j}}, u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{k}}\}$ chosen from $\left]\frac{n}{4}\right]$ sub partitions dominate $2\left(\left]\frac{n}{4}\right] 2(2k-6) - (2k-6)\right) - \frac{n}{2}(2k-12)$ vertices of S^{**}

$\Rightarrow \left|N(u_{2\mathrm{i}-1} v_{\mathrm{j}}) \cup N(u_{2\mathrm{i}-1} v_k) \cup N(u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{j}}) \cup N(u_{2\mathrm{i}-1} \mathrm{e}_{\mathrm{j}})\right| = |S^{**}|$.

Similar number of vertices chosen from S^{**} dominate S^{*} with cardinality $4\,]\frac{n}{4}]$.

From both the cases ,

$D = \{(a, b)/\ a \subset D_1 , b \subset D_2\}$ is the required minimum dominating set for G^*

$\Longrightarrow \gamma(G^*) = 8\,]\frac{n}{4}]$.

IV. CONCLUSION

In this paper , we have discussed and derived the general formula for even cycle graph G with $G^{+--}, G^{+-+}, G^{--+}$ and G^{++-}. Also some results were discussed.

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